Interactions among Overlays and Traffic Engineering: Equilibrium and Cooperation without Payment

Shan Jiang[1], Jun Gong[1], Jingyu Wang[1], Jianxin Liao[1], Tonghong Li[2]

[1]State Key Lab of Networking and Switching Technology

Beijing University of Posts and Telecommunications, Beijing, China

[2]Department of Computer Science

Technical University of Madrid, Madrid, Spain

[1]{jiangshan, wangjingyu, liaojx}@bupt.edu.cn, [1]gongjunonly@gmail.com, [2]tonghong@fi.upm.es

***Abstract*—Emerging overlay technologies have enabled to distribute content efficiently over the Internet that in some sense improves the user quality of experience. However, due to inconsistent or even conflicting objectives from the perspectives of multiple co-existing overlay networks and traffic engineering (TE), the interaction among them impacts the performance of each other and results in sub-optimum, which also has not yet been investigated in detail. In this paper, we model this interaction as an n+1-person non-cooperative game, prove the existence of Nash equilibrium point (NEP) and propose an algorithm to compute NEP. In order to overcome the inefficiency of NEP, we define a non-transferable utility (NTU) game based on Shapley NTU game theory, in which overlays and TE share cost efficiently and fairly without side payment, and we propose an algorithm to calculate the Shapley NTU value (SNTUV).**

Keywords—overlay; traffic engineering; game theory; Nash equilibrium; cost sharing; Shapley NTU value

I. INTRODUCTION

Over the past few years, a wide variety of overlay networks (such as CDN, P2P) have been deployed upon the Internet because their routing strategies can improve the performance of traditional IP layer routing. However, this causes the interaction not only between the overlay and underlay network but also among multiple overlay networks. On the one hand, traditional traffic engineering (TE) techniques, such as Open Shortest Path First (OSPF) and Multiple Protocol Label Switching (MPLS), are adopted by network operators to optimize the global cost of the network, yet the emergence of overlays reallocates traffic from their underlay physical routes for their own objectives thus increasing the cost of TE [1]-[4]. On the other hand, when multiple overlays coexist in the same physical network sharing some physical links and nodes, each overlay performs selfish routing strategy to improve its performance regardless of the influence on others, thus increasing delays of other overlays due to competing for physical network resources [5]-[9]. In particular, these interactions affect the stability and optimality of the global network. The interactions between overlay networks and TE are shown schematically in Fig. 1. Researchers have explored several approaches to solve the inherent conflicts in network environments, such as pricing, bargaining or other incentive mechanism as shown in [3]-[13], all of which, however, have limitations. In this paper, we propose an endogenous scheme for overlays and TE to share cost without side payment and reach the efficient point.

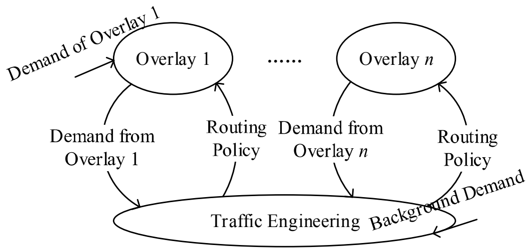

Fig. 1. Interactions among overlays and TE

This paper focuses on the interaction between multiple co-existing overlay networks and TE. We demonstrate the misalignments between TE and overlays that result in the loss of efficiency, and then propose a way of cooperation in order to reach the efficient point without side payment. We try to answer the following questions: Does equilibrium exist in the interaction process? If it exists, does the dynamic interaction converge to the equilibrium point, and does the order of adjusting strategies matter? Is the equilibrium efficient? If not, can multiple co-existing overlay networks and TE cooperate to reach an efficient point? How do they share cost?

Our paper makes the following main contributions:

- We formulate the routing strategies of multiple co-existing overlays and TE under an optimization framework, and show misalignments of objectives between overlay networks and TE.
- Based on the framework, we model the interaction of overlay networks and TE as an n+1-person non-cooperative game, point out the existence of Nash equilibrium point (NEP) and propose two algorithms to compute NEP.
- We define a non-transferable utility (NTU) game to overcome the inefficiency of NEP. Taking into consideration of both Pareto efficiency and fairness, we propose a cost-sharing policy without any side payments based on the Shapley NTU value (SNTUV), and propose an algorithm to calculate SNTUV.

II. RELATED WORK

Game theory was summarized by Osborne [14], and its application in networks environments has been extensively studied. In the area of networking, the user equilibrium modeling the interaction between users as standard network optimization problem was proposed by Roughgarden [15]. Liu et al. [1] studied the interaction between TE and one single overlay by using the best-reply dynamics and demonstrated the impact of the overlay on the underlay network. Jiang et al. [5] studied the interaction between overlays and discovered some implications due to the interaction. Ma and Misra [16] studied the role of congestion in network equilibrium in detail. In order to solve the inherent conflict in this interaction, researchers have explored several schemes. Gong et al. [4] adopted repeated game to reduce the conflicts between overlays and TE. Jiang et al. [5] used a pricing mechanism to improve the performance of overlays. Wang et al. [7] studied the collaborations of multiple selfish overlays by using multi-path resources. Some other studies such as [8] and [9] dealt with similar problems. Applying cooperative game theory in network environments is an alternative way to overcome the inefficiency of NEP. Jiang et al. [3] and Cui et al. [6] adopt Nash bargaining theory to solve this problem. However, Nash bargaining can only be applied to the game with two players, when there are more players in the game, the problem becomes much more complex. Ma et al. [10], Niyato et al. [11] and Misra et al. [12] applied Shapley value to network environments. However, primitive Shapley value requires side payments thus bringing transaction cost to the system.

Our work differs from the previous studies in the following aspects. First, our work focuses on the interaction between TE and multiple co-existing overlays, which is a combinational scenario of [1]-[4] and [5]-[9]. Second, we are the first to propose an endogenous cost-sharing policy without side payment to overcome the efficiency loss during the interaction between overlay networks and TE based on SNTUV theory which was presented by Shapley [17].

III. Model and Problem Statement

In this section we formally model the co-existing overlay networks and TE. We introduce the notations and objectives of both overlay routing and TE, and then use an example to illustrate the framework.

A. Network Model

The underlay network provides physical links for demands from overlay and background users. Let $G=(V,E)$ represent the underlay network topology, where V is the set of physical nodes, E is the set of physical links. Then we define a capacity vector $C=(C_{e_1},C_{e_2},\cdots,C_{e_{|E|}})$, where C_e is the capacity for each link $e\in E$, a routing set R, where each route $r\in R$ denotes a possible route of the underlay network, and a $|E|\times|R|$ physical indicator matrix A where $a_{er}=1$ if route r traverses link e and $a_{er}=0$ otherwise.

An overlay s in the logical level is represented by graph $G^{(s)}=(V^{(s)},E^{(s)})$, where $V^{(s)}$ is the set of logical nodes and $E^{(s)}$ is the set of logical links. Each overlay node maps onto a physical node, and each overlay link maps onto a set of physical routes. A logical path $r^{(s)}\in R^{(s)}$ contains a set of logical links. Each overlay may have several *demands*, each of which is a source-sink pair associated with a flow f with traffic volume x_f. Consider there are n overlays on top of an underlay network and let N denote the full set of overlays, thus $s\in N$. Let F_s denote all demands of overlay s. Also, we consider background demands b from underlay users that directly use the underlay network to transfer data. Let F_b represent all background demands. We use set $F=\cup_{s\in N}F_s\cup F_b$ to denote all flows. Let $b^{(s,f)}(b^{(b,f)})$ denote a $|E^{(s)}|\times|R_f^{(s)}|$ $(|E^{(b)}|\times|R_f^{(b)}|)$ logical indicator matrix, where $b_{er}^{(s,f)}=1$ $(b_{er}^{(b,f)}=1)$ if flow f traverses logical link $e^{(s)}$ $(e^{(b)})$ and $b_{er}^{(s,f)}=0$ $(b_{er}^{(b,f)}=0)$ otherwise. Then, for completeness, we can rewrite the logical indicator matrix B as follows: $B=(b^{(1)},\cdots,b^{(n)},b^{(b)})^T, b^{(i)}=(b^{(i,f_1)},\cdots,b^{(i,f_{|F_i|})}), i\in N,b$.

The overlay determines the routing policies for all demands in its logical network. Source node may divide its demand and allocate onto different paths. For each flow $f\in F_s$, the overlay needs to determine how to allocate its volume x_f to possible paths. i.e., its volume allocation decision vector for flow f is $y^{(s,f)}=(y_{r_1^{(s)}}^{(s,f)},y_{r_2^{(s)}}^{(s,f)},\cdots,y_{|R_f^{(s)}|}^{(s,f)})^T$, where $y_{r^{(s)}}^{(s,f)}$ is the traffic volume on logical path $r^{(s)}$ for flow f in overlay s. We have $\sum_{r^{(s)}\in R_f^{(s)}}y_{r^{(s)}}^{(s,f)}=x_f, f\in F_s$. Similarly, $y^{(b,f)}$ is the decision defined for background flow $f\in F_b$. As no routing policy is applied for those background flows, we have $y^{(b,f)}=x_f, f\in F_b$. We rewrite matrix Y as: $Y=\left(y^{(1)},\cdots,y^{(n)},y^{(b)}\right), y^{(i)}=(y^{(i,f_1)},\cdots,y^{(i,f_{|F_i|})})^T, i\in N,b$.

The overlay and background users pass their demands to the underlay. And TE decides how to allocate the traffic on physical links. The total volume between two neighbor logical nodes in fact maps onto a physical demand, i.e., $\sum_{i\in N,b}e^{(i)}$ corresponds to demands from the logical source node of $e^{(i)}$ to the sink node. Denote Z as a $|R|\times\sum_{i\in N,b}|E^{(i)}|$ matrix and its element $z_{re^{(s)}}$ is the fraction of volume from logical links that TE allocates to route r, and we have $\sum_{r\in R_{e^{(s)}}}z_{re^{(s)}}=1$ for each flow logical link. Here TE does not differentiate demands of overlay users and underlay users and performs the same fractions for demands with the same source-sink. Then the volume on each link e: $L=(l_{e_1},l_{e_2},\cdots l_{e_{|E|}})^T$ is:

$$L=AZ\sum_{i\in N,b}\sum_{f\in F}b^{(i,f)}y^{(i,f)}=AZ\sum_{i\in N,b}b^{(i)}\,y^{(i)}=AZBY, \quad (1)$$

where l_e is the total volume of demands allocated onto link e.

We call allocation decisions Y,Z of overlays and TE *feasible* if they satisfy the conditions that $Y\geq 0, Z\geq 0, L^T\leq C$. i.e., the volume on logical links and fraction on physical links are non-negative and the aggregate volume allocated to link e is no more than its capacity C_e.

1) Objective of TE

The objective of TE is to minimize the overall cost in the whole physical network, in terms of congestion, maximum link utilization and delay, etc. In this paper, we assume that the cost of TE is denoted by the total delay of all users in overlays and underlay. Denote $d_e(l_e)$ as the unit delay function for link $e\in E$, and the delay function is continuous, increasing and convex. For all links, their delays are represented by a unit delay vector: $D(L)=(d_{e_1}(l_{e_1}),d_{e_2}(l_{e_2}),\cdots,d_{e_{|E|}}(l_{e_{|E|}}))^T$.

The delay of the whole physical network is:

$$cost^{TE} = \sum_{e \in E} l_e \cdot d_e(l_e). \tag{2}$$

The optimization problem for TE can be rewritten as:

$$\min cost^{TE}(Z) = L^T D(L)$$
$$s.t. \begin{cases} \forall e^{(s)} \in E^{(s)}, \sum_{r \in R_{e^{(s)}}} z_{re^{(s)}} = 1, \\ Y \geq 0, Z \geq 0, L^T \leq C \end{cases} \tag{3}$$

where TE considers the flow strategy Y from overlays as *parameter*, and its fraction allocation Z as the *variable*.

2) Objective of Overlays

The objective of the overlay is to minimize the delay in its logical network. By using the notation we defined, the delay for each overlay s is:

$$delay^s = \sum_{e^{(s)} \in E^{(s)}} l_{e^{(s)}} \sum_{e \in r, r \to e^{(s)}} z_{re^{(s)}} d_e(l_e), \tag{4}$$

where $l_{e^{(s)}}$ represents the volume on logical link $e^{(s)} \in E^{(s)}$ caused by overlay s.

Here we expand the size of $b^{(s)}$ and $y^{(s)}$ to B and Y, respectively, by filling the vacant elements with zero. Thus the optimization problem for each overlay s can be written as:

$$\min delay^s(y^{(s)}) = (AZb^{(s)}y^{(s)})^T D(L)$$
$$s.t. \begin{cases} \forall f \in F_s, \sum_{r^{(s)} \in R_f^{(s)}} y_{r^{(s)}}^{(s,f)} = x_f \\ Y \geq 0, Z \geq 0, L^T \leq C \end{cases}, \tag{5}$$

where overlay s considers the fraction allocation of TE Z and the flow strategy from other overlays $y^{(-s)}$ as *parameters*, and its flow strategy $y^{(s)}$ as the *variable*.

B. Example

In order to illustrate the notations and objectives above, we use a network in Fig. 2 as an example. The physical network is a 6-node directed graph. There are two co-existing overlays deployed on the underlay network. The strategies of overlays and TE are shown. We assume that overlay 1 has 1 unit demand from B_1 to D_1, and overlay 2 has 1 unit demand from C_2 to F_2, therefore the total volume of flow on logical path $B_1 - A_1 - D_1$ is $y_{B_1-A_1-D_1}^{(1,1)}$, and on path $B_1 - E_1 - D_1$ is $y_{B_1-E_1-D_1}^{(1,1)} = 1 - y_{B_1-A_1-D_1}^{(1,1)}$. Besides, we assume there is 1 unit background demand between each neighbor overlay nodes. Note that each logical link between two neighbor overlay nodes maps onto a set of physical routes, and TE can allocate the fraction of flow on the physical networks, for example, the fraction on $A - D$ is z_{A-D,A_1-D_1} and on $A - C - D$ is $z_{A-C-D,A_1-D_1} = 1 - z_{A-D,A_1-D_1}$. Now we consider the total volume in each physical link e, which is the sum of demands from all logical layers. For example, there are 4 demands on CD: $y_{B_1-A_1-D_1}^{(1,1)}$ from overlay 1, $y_{C_2-F_2}^{(2,1)}$ from overlay 2, and 1 unit from underlay $A - D$ and $C - F$ respectively, thus the total volume on link CD is: $l_{CD} = z_{C-D-F,C_2-F_2}(1 + y_{B_1-A_1-D_1}^{(1,1)}) + z_{A-C-D,A_1-D_1}(1 + y_{C_2-F_2}^{(2,1)})$.

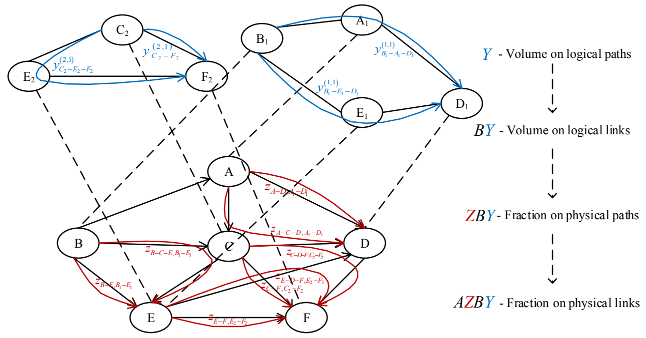

Fig. 2. An example of overlay networks and TE

IV. N+1-PERSON NON-COOPERATIVE GAME

In this section, we model the interaction between multiple overlays and TE as an n+1-person non-cooperative game. We build the model of this game then propose two distinctive methods to compute NEP.

A. Non-Cooperative Game

We now define the non-cooperative game between overlays and TE. First, in a finite set of players $N + 1 = \{1,2,\cdots,n+1\}$, the first n players are overlays and the last player is TE. Second, the strategy of an overlay s is a volume matrix of demands among all available paths, and the strategy of TE is a fraction allocation matrix for the whole networks. Furthermore, the strategy should be *feasible*.

The set of profiles of TE and overlays is described in (6).

$$\Gamma_i = \begin{cases} \left\{ y^{(i)} \middle| \begin{array}{l} y^{(i)} \in \mathbb{R}_+^{|R^{(i)}| \times \sum_{f \in F_i} |R_f^{(i)}|}, \\ Y \geq 0, L^T \leq C, \forall f \in F_i, \\ \sum_{r^{(i)} \in R_f^{(i)}} y_{r^{(i)}}^{(i,f)} = x_f \end{array} \right\}, i = 1, \cdots, n \\ \left\{ Z \middle| \begin{array}{l} Z \in \mathbb{R}_+^{|R| \times \sum_{j \in N, b} |E^{(j)}|}, \\ Z \geq 0, L^T \leq C, \forall e^{(s)} \in E^{(s)}, \\ \sum_{r \in R_{e^{(s)}}} z_{re^{(s)}} = 1 \end{array} \right\}, i = n+1 \end{cases}, \tag{6}$$

where $\mathbb{R}_+$ denotes the set of nonnegative real number. Then, we define a preference relation $\succsim_i$ for player i. Here we say $y^{(s)} \succsim y^{(s)\prime}$ if $delay^s(y^{(s)}) \leq delay^s(y^{(s)\prime})$. Similarly, we say $Z \succsim Z'$ if $cost^{TE}(Z) \leq cost^{TE}(Z')$. Finally, we define the routing game as $GM\langle N+1, (\Gamma_i), (\succsim_i)\rangle$ and its NEP is defined as follows.

Definition 1. A strategy profile $y^* \in \Gamma_1 \times \cdots \times \Gamma_n, Z^* \in \Gamma_{n+1}$ is NEP if the profile is feasible and $\forall i \in N$:

$$\begin{array}{l} \forall y'^{(i)} \in \Gamma_i, \\ delay^i(y^{*(i)}, y^{*(-i)}, Z^*) \leq delay^i(y'^{(i)}, y^{*(-i)}, Z^*). \\ \forall Z' \in \Gamma_{n+1}, \\ cost^{TE}(Y^*, Z^*) \leq cost^{TE}(Y^*, Z') \end{array} \tag{7}$$

Namely, NE describes a situation where no player can improve its own objective by altering its routing strategy unilaterally. NE is a stable state, since all plays do not have inducements to change their strategies.

Theorem 1. In $GM\langle N+1,(\Gamma_i),(\succsim_i)\rangle$, NEP exists if $cost^{TE}$ function and $delay^s$ function are continuous, non-decreasing, and strictly convex.

Proof: Rosen [18] proves that a game has NEP if the game meets the following conditions: (1) The set of profile Γ_i is a nonempty compact convex subset of a Euclidean space. (2) The preference relation $\succsim_i$ is quasi-concave and continuous on Γ_i. Firstly, the strategy profiles in $GM\langle N+1,(\Gamma_i),(\succsim_i)\rangle$ are well defined by the capacity of links and the non-negativity constraint $Y \geq 0, Z \geq 0, L^T \leq C$ with a closed and bounded feasible region, thus Γ_i is compact. Moreover, all constraints are affine functions and the feasible domain is the intersection of half-spaces and hyperplanes, thus Γ_i is convex. Therefore GM fits the first condition. Secondly, the delay function on each link $e \in E$ is continuous, thus the preference relation $\succsim_i$ is also continuous. Since $cost^{TE}$ and $delay^s$ functions that are the negative of payoff functions are convex, the payoff function is concave, and preference relation is quasi-concave.■

In order to compute the NEP allocation, we define the notation of *best response*.

Definition 2. Given $GM\langle N+1,(\Gamma_i),(\succsim_i)\rangle$, player i's best response to the strategies of other players is the strategy that minimizes its delay or cost function, which is:

$$\begin{aligned}&y^{(i)}(y^{(-i)},Z) = \operatorname{argmin} delay^i(y^{(i)},y^{(-i)},Z), i=1,\cdots n\\&Z(Y) = \operatorname{argmin} cost^{TE}(Z,Y), i=n+1\end{aligned}. \quad (8)$$

By Definition 1, we know NEP is the result when each player performs its best response strategy. Furthermore, quasi-concavity of each player's objective function implies the uniqueness of the best response thus the stability of NEP, which, however, does not imply the uniqueness of NEP, even though dynamic response converges to NEP. In general, NEP may be multiple.

Here we compute NEP by two approaches.

B. Simultaneous Best Response

We first write out the best response function of each player, and then solve equations to obtain NEP. The best response in this situation was studied by Roughgarden [15]. In their work, Karush-Kuhn-Tucker (KKT) conditions are used to solve the problem, in which all routes with non-zero volume serving the same demand must have the same end-to-end *length*, and the length of a link is interpreted as the first derivative of *its* volume multiplies unit delay at current level. We modify this concept of KKT conditions into our work, where the length is $\partial(l_e \cdot d_e(l_e))/\partial z_{re^{(s)}}$ for each physical link e and $\partial(l_{e^{(s)}} \cdot \sum z_{re^{(s)}} d_e(l_e))/\partial y^{(s,f)}_{r^{(s)}}$ for each logical link $e^{(s)}$, and the length of a route r or $r^{(s)}$ is the summation of all physical or logical links that the route traverses. Furthermore, if the length of a route is the minimum among all available routes serving the demand, volume is assigned on this route.

The KKT conditions for TE are:

$$\forall r \in R, \frac{\partial[\sum_{e\in r} l_e \cdot d_e(l_e)]}{\partial z_{re^{(s)}}} \begin{cases} = u_i, \text{if } z_{re^{(s)}} > 0 \\ \geq u_i, \text{if } z_{re^{(s)}} = 0 \end{cases}. \quad (9)$$

The KKT conditions for overlay s are:

$$\forall r^{(s)} \in R^{(s)}, \frac{\partial\left[\sum_{e^{(s)}\in r^{(s)}} l_{e^{(s)}} \sum_{e\in r, r\to e^{(s)}} z_{re^{(s)}} d_e(l_e)\right]}{\partial y^{(s,f)}_{r^{(s)}}} = \begin{cases} = u_i, \text{if } y^{(s,f)}_{r^{(s)}} > 0 \\ \geq u_i, \text{if } y^{(s,f)}_{r^{(s)}} = 0 \end{cases}, \quad (10)$$

where u_i refers to the minimum end-to-end first derivative length for the demands.

If players want to perform equilibrium strategies, they need to have *perfect information* of each other, which means every player undoubtedly believes that everyone else in this game will perform equilibrium strategies, otherwise their strategies are meaningless. This hypothesis is too strict in the real world since normally players need time to adjust their strategies.

C. Dynamic Best Response

We now propose the dynamic approach that converges to NEP, which means players do not possess perfect information at the beginning of the game, and they perform their strategies based on the existent situation. Since players repeatedly interact with each other, they will gradually exchange information of others and finally have the perfect information, which leads to the convergence of NEP.

In order to compute dynamic best response, first, we give TE and overlay networks an *initial allocation*, which is an arbitrary feasible value of Y, Z. Then, TE and overlays take turns to optimize their allocations repeatedly until they reach NEP. The algorithm is demonstrated as follows:

Algorithm 1. Computing NEP $(G, G^{(s)}, F, A, B, C, D)$**:**
1. Set an initial allocation $Z(0), Y(0)$ and $t = 1$;
2. **do**
3. TE alters $Z(t)$ to optimize its objectives;[1]
4. Overlay 1 alters $y^{(1)}(t)$ to optimize its objectives;
5. ……
6. Overlay n alters $y^{(n)}(t)$ to optimize its objectives;
7. $t = t + 1$;
8. **until** $Z(t) = Z(t-1), Y(t) = Y(t-1)$

Here the sequence of adjusting strategies can vary.

V. COOPERATION WITHOUT PAYMENT

It has been pointed out that NEP is usually inefficient [15]. Therefore, the performance improvement is possible. One way to reach an efficient point is through cooperation. In this paper, we adopt SNTUV among all cooperation mechanisms to assign cost for the following reasons:

- SNTUV can apply to game with multiple players, which generalizes Nash bargaining value.
- SNTUV is an endogenous mechanism thus causing no transaction cost, which generalizes Shapley value.

A. Shapley TU Game

In order to obtain SNTUV, we begin with introducing transferable utility (TU) game. When there are n+1 players and the payoff is defined by cost (all players trying to minimize their objectives), a TU game is a pair $(N+1, v)$ [19], where

[1]Some researches such as [1] show that this interaction may not converge to NEP, the reason is that players may randomly alter between strategies with equal delay, Here we stipulate that a player alters its strategy *only when* it gets less delay by using another strategy, otherwise, it keeps strategy invariable.

$N+1$ is the set of players and v is the *coalitional function*. For a TU game, there exists a *real number* $v(S)$ which is:

$$v(S) = \inf \left\{ \sum_{i \in S} \tau_i : \tau \in \mathbb{R}^S \right\}, \tag{11}$$

(In our work, the infimum is a minimum.) where $\tau = (\tau_i)_{i \in S}$ is the objective cost for the coalition S whose utility for player i is defined by $delay^i, i = 1,2, \cdots n, cost^{TE}, i = n+1$, and the Shapley TU value for $(N+1, v)$ is computed by:

$$\varphi_i^{TU}(v) = \sum_{S: i \notin S} \frac{|S|!\,(n-|S|)!}{(n+1)!} [v(S \cup i) - v(S)]. \tag{12}$$

Note that the cost for each player in network environments is inherently non-transferable (players cannot directly transfer the cost to others), thus this kind of cost-sharing mechanism may not be feasible, thus needing side payment as intermediary and external coordination which may bring transaction cost. Therefore, we use SNTUV to avoid these problems.

B. Shapley NTU Game

NTU game is a generalization of TU game. An NTU game in *coalition form* is denoted by a pair $(N+1, V)$, where V is an association of each coalition $S \subset N+1$ and the set $V(S)$ of feasible payoff vector for S. In the NTU game, $V(S)$ is the set of *cost combinations* that is feasible for the coalition S. In an NTU game, we use a comparison weight $\lambda \in \mathbb{R}_+^{N+1}, \lambda \neq 0$ to *unify* the coalition function which is:

$$v_\lambda(S) = \inf \left\{ \sum_{i \in S} \lambda_i \tau_i : \tau \in V(S) \right\}, \tag{13}$$

and SNTUV for $(N+1, V)$ is computed by:

$$\varphi_i^{NTU}(V) = \frac{\varphi_i^{TU}(v_\lambda)}{\lambda_i}, \tag{14}$$

where $\varphi_i^{NTU}(V)$ is feasible for all $i \in N+1$.

SNTUV satisfies the following axioms, which are proved by Aumann [20].

Axiom 1. Non-emptiness: $\varphi^{NTU}(V) \neq \emptyset$, where $\emptyset$ is the empty set.

Axiom 2. Efficiency: $\varphi^{NTU}(V) \subset \partial V(N)$, where $\partial V(N)$ is the Pareto efficient boundary of $V(N)$.

Axiom 3. Fairness (Unanimity): if U_s is the unanimity game on a coalition S, then there is: $\varphi^{NTU}(U_s) = \{|S|^{-1}\}$.

Axiom 4. Conditional Additivity: if $U = V + W$, then there is: $\varphi^{NTU}(U) \supset (\varphi^{NTU}(V) + \varphi^{NTU}(W)) \cap \partial U(N)$.

Axiom 5. Closure Invariance: $\varphi^{NTU}(\bar{V}) = \varphi^{NTU}(V)$.

Axiom 6. Scale Covariance: if $\lambda \in \mathbb{R}_+^{N+1}, \lambda \neq 0$, then there is: $\varphi^{NTU}(\lambda V) = \lambda \varphi^{NTU}(V)$.

Note that *efficiency* demonstrates that all SNTUV are Pareto efficient, and *fairness* demonstrates that the unanimity game on S has a unique value, which implies that coalition S assigns the amount of cost for each player fairly.

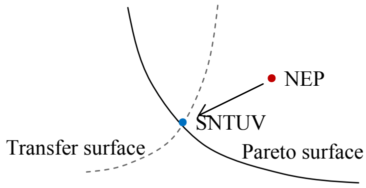

Fig. 3. Illustration of SNTUV

We illustrate SNTUV schematically in Fig. 3, where the Pareto surface is a set of feasible efficient allocations and the transfer surface is a set of Shapley TU values. We view the problem of TE and overlays as a combinatorial optimization, where the Pareto surface indicates the least cost combination for all players. Then we know NEP is inefficient thus not on the Pareto surface. If we directly use the summation of the cost for each player as the coalition function, the Shapley value may deviate from Pareto surface due to the non-transferability of cost. Therefore, we unify the coalition function by λ and recalculate the Shapley value until find a feasible allocation which is SNTUV. Now we propose the existence of SNTUV:

Theorem 2. In a NTU game $(N+1, V)$, for each $S \subset N+1$, there exists $\varphi_i^{NTU}(V) \in V(S)$ such that $\lambda_i \varphi_i^{NTU}(V) = \varphi_i^{TU}(v_\lambda)$.

Proof: We prove this theorem briefly in Fig. 3. By using different sets of weight λ, the position of the Shapley value changes as weights vary, thus constituting the transfer surface. Since the Pareto surface and transfer surface are unbounded and unparallel, the intersection point of two surfaces invariably exists. Namely, the SNTUV invariably exists (although may not be unique). Detailed proof was proposed in [20]. ■

The algorithm to compute SNTUV is shown as follows:

Algorithm 2. Computing SNTUV $(G, G^{(s)}, F, A, B, C, D)$**:**
1. **for** each weight vector $\lambda \in \mathbb{R}_+^{N+1}, \lambda \neq 0$
2. **for** each coalition circumstance (none, partial, grand)
3. Find NEP and obtain $v_\lambda(S)$ for each S;
4. Calculate $\varphi_i^{TU}(v_\lambda)$ for each i by using (12);
5. Calculate $\varphi_i^{NTU}(V)$ for each i by using (14);
6. **if** φ_i^{NTU} is feasible for all $i \in N+1$
7. **then** stop algorithm;

In step 3, first, when players do not form any coalition, the result is NEP. Second, when players form partial coalition, each partial coalition has an objective function which is the summation of each member's cost multiplies the corresponding weight λ, furthermore, each coalition or single player has its objective function, and thus we need to recalculate NEP in view of this circumstance. Third, when players form grand coalition, the result is Pareto optimal.

Some difficulties may exist in the allocation cooperation mechanism. First, SNTUV may not be stable if the player obtains more delay in the grand coalition than in the partial coalition, or even in Nash equilibrium. The conception for stability is called core and is proposed in [21]. Second, computing Shapley value is fundamentally complex [12] and computing SNTUV is even more so due to the variability of λ. Provided we eliminate the potential problems, SNTUV is a satisfied allocation.

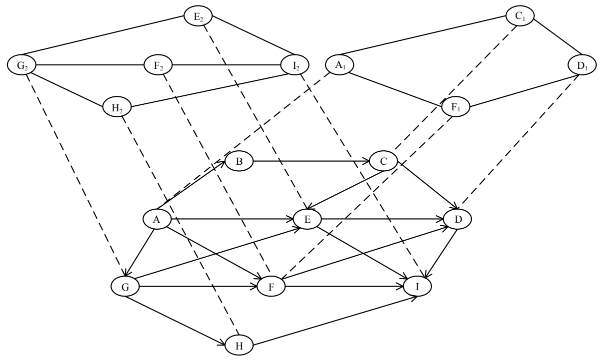

Fig. 4. Simulation network topologies

VI. Simulation and Evaluation Results

In this section, we conduct a simulation based on the network topologies demonstrated in Fig. 4, which contain a 9-node underlay network and two overlay networks deployed above sharing some common physical links. In order to compute NEP, we adopt an M/M/1 queuing delay function: $d_e(l_e) = 1/(C_e - l_e)$ and set the capacity of link $C_e = 3$ and $C_e = 5$ for different links respectively.

A. Interaction Converging to NEP

Initially, we set TE to adopt the shortest route to transmit traffic volume and set an equalization allocation to overlay networks, which means all flows are equally allocated on all possible logical paths. Since there are 3 players, we test all six possible sequences using best response method. The simulation results are shown in Fig. 5-7 (overlay denoted by OR), where each iteration represents the turn of any player. The convergence yields the results of NEP which is: $cost^{TE} = 7.763$, $delay^1 = 1.717$, $delay^2 = 1.379$.

We observe that there are some implications during the iteration process. At the beginning of the process, the condition of the network is relatively terrible and the interaction among overlays and TE is mutually beneficial, because the misalignments have not unveiled yet. However, the conflicts began to reveal due to their different objectives as the iteration proceeds, and we can see some oscillations in the middle of each figure. Eventually, the oscillation subsides and come to a stable allocation which is NEP. We also find that the different sequence of adjusting one's strategy does not influent the ultimate stability. In our experiment, all possible sequences converge to the same NEP. However in general, the results may vary due to the multiplicity of NEP. Nevertheless, the convergence is inevitable after certain iterations, which leads to a stable NEP.

B. Cooperation to Obtain SNTUV

We now compute the SNTUV of the network topologies. Since there are three players, we need to consider both partial and grand coalition. There is one grand coalition and three possible partial coalitions, i.e., $\{OR1, OR2, TE\}$, $\{OR1, OR2\}$, $\{OR1, TE\}$ and $\{OR2, TE\}$. When players form the coalition, their objectives are unified by the weight vector, thus changing the resulting Shapley value. Since only ratio between weights matters in the end, we set $\lambda_3 = 1$ and find the suitable weights of others. Computing different weights obtains a set of possible SNTUV. In order to verify its feasibility, we need to calculate the Pareto surface by using the multiobjective particle swarm optimization (MOPSO) proposed in [22] to obtain all efficient points. The simulation results are shown in Fig. 8, where Pareto surface (marked by black ×) denotes the Pareto efficient points, transfer surface (marked by grey +) denotes the possible SNTUV, and their intersection points are the SNTUV (marked by blue point). In order to compare results, we also plot NEP (marked by red point).

As the iteration proceeds, a vector λ=(0.878,9.575,1.000) yielding the feasible solution was found. The SNTUV is: $cost^{TE} = 7.727$, $delay^1 = 1.708$, $delay^2 = 1.378$. And the coalition functions are described as follows:

$$\begin{aligned} &V(\{OR1\}) = \{\tau_1 : \lambda_1\tau_1 \geq 1.508\} \\ &V(\{OR2\}) = \{\tau_2 : \lambda_2\tau_2 \geq 13.202\} \\ &V(\{TE\}) = \{\tau_3 : \tau_3 \geq 7.763\} \\ &V(\{OR1, OR2\}) = \{(\tau_1, \tau_2) : \lambda_1\tau_1 + \lambda_2\tau_2 \geq 14.865\} \\ &V(\{OR1, TE\}) = \{(\tau_1, \tau_3) : \lambda_1\tau_1 + \tau_3 \geq 9.183\} \\ &V(\{OR2, TE\}) = \{(\tau_2, \tau_3) : \lambda_2\tau_2 + \tau_3 \geq 20.953\} \\ &V(OR1, OR2, TE) = \{(\tau_1, \tau_2, \tau_3) : \lambda_1\tau_1 + \lambda_2\tau_2 + \tau_3 \\ &\qquad \geq 22.416\} \end{aligned} \tag{15}$$

The deviation between NEP and Pareto surface implies the inefficiency of NEP because one player can reduce its cost without increasing the cost of others. When varying λ we get a set of possible Shapley value thus constituting the transfer surface. Here are several intersection points, however, some of them are bad for certain players than NEP. (Note that some Pareto efficiency may not improve performance for all players.) Therefore, we choose between the possible results and get the satisfied SNTUV.

We now compare the cost of TE and overlay networks, and the results are demonstrated in Fig. 9, where each subfigure represents the cost of TE or delay of overlay respectively. We observe that all TE and overlays can improve their performance

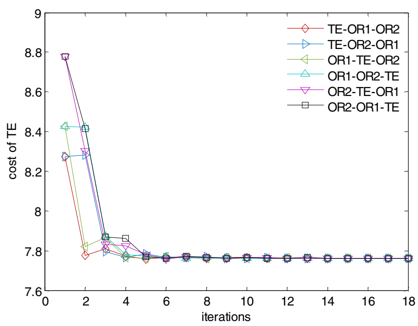

Fig. 5. Cost of TE over interactions

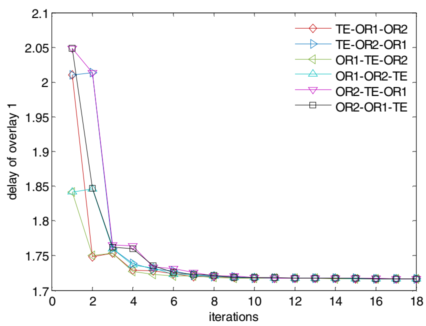

Fig. 6. Delay of overlay 1 over interactions

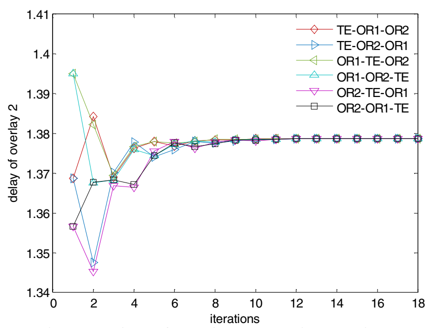

Fig. 7. Delay of overlay 2 over interactions

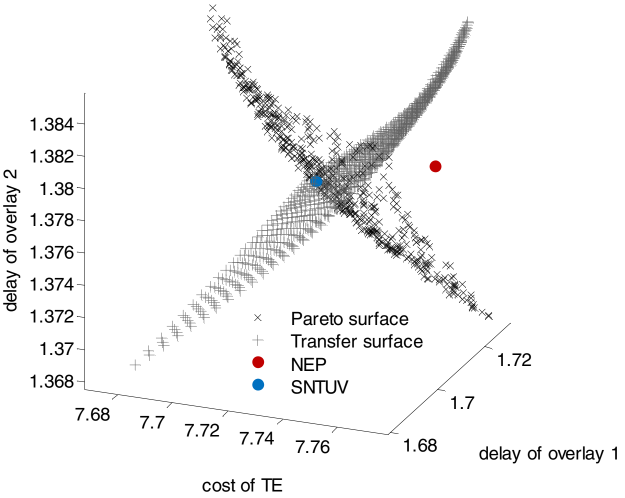

Fig. 8. Simulation results of NTU value

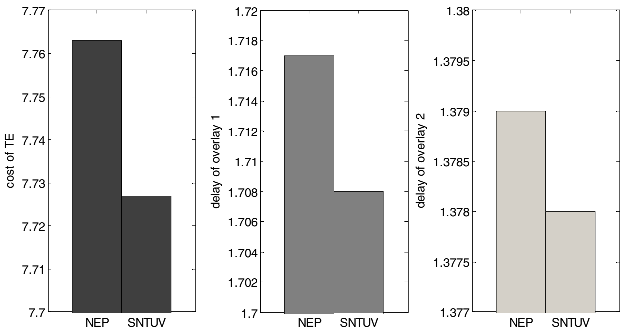

Fig. 9. Cost and delay comparison of NEP and NTU value

without any side payment. Moreover, the amount of cost they reduced is assigned fairly according to Shapley's theory. Note that SNTUV requires all players deviating from NEP thus needing cooperation among all players.

VII. Conclusion

This paper studies the interaction between multiple overlays and TE by using both non-cooperative and cooperative game theoretic approach. We demonstrate the existence and inefficiency of NEP and how the interaction converges to the point. We also adopt SNTUV to improve the performance of the network and assign cost. However, the calculation of SNTUV is still an open issue, which needs further investigation.

Acknowledgment

This work was jointly supported by: (1) the National Basic Research Program of China (No. 2013CB329102); (2) National Natural Science Foundation of China (No. 61471063, 61421061, 61372120, 61271019, 61101119, 61121001); (3) the Key (Keygrant) Project of Chinese Ministry of Education. (No. MCM20130310); (4) Beijing Municipal Natural Science Foundation (No. 4152039); (5) Beijing Higher Education Young Elite Teacher Project (No. YETP0473); (6) Spanish Research Council (No: TIN2013-46883); (7) Regional Government of Madrid (No: S2013/ICE-2894) cofunded by FSE & FEDER.

References

[1] Y. Liu, H. Zhang, W. Gong, and D. Towsley, "On the interaction between overlay routing and underlay routing," in Proc. IEEE INFOCOM, pp. 2543–2553, 2005.

[2] J. Xiao, R. Boutaba, "Reconciling the Overlay and Underlay Tussle," Networking, IEEE/ACM Transactions on., vol.22, no.5, pp.1489,1502, Oct. 2014.

[3] W. Jiang, R. Zhang-Shen, J. Rexford, M. Chiang, "Cooperative content distribution and traffic engineering in an ISP network," ACM SIGMETRICS Performance Evaluation Review. vol. 37. no. 1., 2009.

[4] J. Gong, J. Liao, J. Wang, Q. Qi, L. Zhang, "Reducing the oscillations between overlay routing and traffic engineering by repeated game theory," Communications (APCC), 2013 19th Asia-Pacific Conference on , vol., no., pp.591,596, 29-31 Aug. 2013.

[5] W. Jiang, D. M. Chiu, and J. C. S. Lui, "On the interaction of multiple overlay routing," Perform. Eval., vol. 62, no. 1-4, pp. 229–246, 2005.

[6] Z. Cui, J. Liao, J. Wang, Q. Qi, J. Wang, "Cooperative overlay routing in a multiple overlay environment," Communications (ICC), 2014 IEEE International Conference on., 2014.

[7] J. Wang, J. Liao, T. Li, J. Wang, "On the collaborations of multiple selfish overlays using multi-path resources," Peer-to-Peer Networking and Applications, vol. 8, No. 2, pp. 203-215, Mar. 2015.

[8] W. Wu, R.T.B. Ma, J.C.S. Lui, "Distributed Caching via Rewarding: An Incentive Scheme Design in P2P-VoD Systems," Parallel and Distributed Systems, IEEE Transactions on , vol.25, no.3, pp.612,621, Mar. 2014.

[9] Cohen, R.; raz, d., "Cost-Effective Resource Allocation of Overlay Routing Relay Nodes," Networking, IEEE/ACM Transactions on , vol.22, no.2, pp.636,646, Apr. 2014.

[10] R. T. B. Ma, D. M. Chiu, J. C. S. Lui, V. Misra, D. Rubenstein, "Internet economics: the use of Shapley value for ISP settlement," IEEE/ACM Trans. Netw. vol. 18, no. 3, pp. 775-787. June 2010.

[11] D. Niyato, P. Wang, E. Hossain, W. Saad, Z. Han, "Game theoretic modeling of cooperation among service providers in mobile cloud computing environments," Wireless Communications and Networking Conference (WCNC), 2012 IEEE , no., pp.3128,3133, 1-4 April 2012.

[12] V. Misra, S. Ioannidis, A. Chaintreau, L. Massouli,. "Incentivizing peer-assisted services: a fluid shapley value approach," In Proceedings of the ACM SIGMETRICS international conference on Measurement and modeling of computer systems (SIGMETRICS '10). ACM, New York, NY, USA, pp. 215-226. 2010.

[13] R.T.B. Ma, J.C.S. Lui, V. Misra, "Evolution of the Internet Economic Ecosystem," Networking, IEEE/ACM Transactions on , vol.23, no.1, pp.85,98, Feb. 2015.

[14] M. Osborne, A. Rubinstein, A course in game theory, MIT press, 1994.

[15] T. Roughgarden and E. Tardos, "How bad is selfish routing?," J. ACM, vol. 49, no. 2, pp. 236–259, 2002.

[16] R. T. B. Ma, V. Misra, "Congestion and Its Role in Network Equilibrium," Selected Areas in Communications, IEEE Journal on, vol.30, no.11, pp.2180-2189, Dec. 2012.

[17] L. S. Shapley, "Utility comparison and the theory of games," The Shapley value: 307, 1967.

[18] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave n-person games," Econometrica: Journal of the Econometric Society, vol. 33, no. 3, pp. 520-534,1965.

[19] S. Hart, "A comparison of non-transferable utility values," Essays in Cooperative Games. Springer US, vol. 36, pp 35-46, 2004.

[20] R. J. Aumann, "An axiomatization of the non-transferable utility value," Econometrica: Journal of the Econometric Society, vol. 53, no. 3, pp. 599-612, 1985.

[21] L. Shapley. "Cores of convex games," International Journal of Game Theory, pp. 11–26, 1971.

[22] C. A. C. Coello, G. T. Pulido, and M. S. Lechuga, "Handling multiple objectives with particle swarm optimization," IEEE Trans. Evol. Comput., vol. 8, no. 3, pp. 256–279, 2004.